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**Interface Practices Subcommittee**

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**AMERICAN NATIONAL STANDARD**

**ANSI/SCTE 58 2017**

**AM CROSS MODULATION MEASUREMENTS**

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## 1.0 PREFACE

This document describes a test procedure for the laboratory and production measurement of Amplitude Modulation Cross Modulation (or AM-XMOD) that is present in Broadband Systems which carry Frequency Division Multiplexed (FDM), amplitude modulated, analog video channels. In general there are two components to the cross modulation generated in such systems; namely, cross modulation of the direct Amplitude Modulation (AM-XMOD) type and cross modulation of the less obvious Phase Modulation (PM-XMOD) type. In the past, the PM-XMOD component was not directly considered, for many pertinent reasons. With the advent of larger cable telecommunications systems (e.g. more video channels with carrier placement at higher frequencies), and the implementation of different equipment and technology (such as fiber optics in the distribution path), it has become necessary that the PM-XMOD at least be described. The physical processes by which the two components of cross modulation are generated have some fundamental differences. However, the resultant video impairment to the viewer can be the same if the proper conditions exist. (That is, both of these components have been shown to cause similar visual disturbances in the desired picture when the proper conditions exist and hence are considered to be true impairments.) **The issue of how each component of cross modulation is detected at the subscriber's terminal (i.e. "set-top receiver") and the subsequent perceptibility levels that should be specified will NOT be addressed in this document.**

The procedure described in this document for measuring the 'AM-XMOD Distortion Ratio' uses an RF Receiver, to detect the modulation, and a Baseband Analyzer, to measure the detected modulation signal levels. The test method, as given, is meant to mitigate the effects that PM-XMOD has upon the accurate detection of AM-XMOD. A common alternative to this measurement technique is to measure the AM-XMOD using a specific instrument, namely the 'Spectrum Analyzer'. The Spectrum Analyzer has the capability to both detect and measure the magnitude of the modulation level (i.e. the levels of the sidebands) in the frequency domain. An AM-XMOD measurement that uses this "frequency domain" technique will be different from a measurement made using the technique described in this procedure, because the Spectrum Analyzer can not discriminate AM and PM sidebands. The type of detector used in the RF Receiver, as well as the method of detection used in the Baseband Analyzer to measure the sideband energy of the modulation, will be specified.

The detectors used in the RF Receiver and the Baseband Analyzer instruments are different than those used in a typical Spectrum Analyzer. Therefore, a Spectrum Analyzer, where used in this document, is understood to be a separate instrument from either an RF Receiver or a Baseband Analyzer. However, it is generally understood that a Spectrum Analyzer can be used as a receiver instrument, whereby either the IF output is taken and RF demodulation is accomplished externally by a separate detector, or the Spectrum Analyzer's (Linear) demodulator is used. In either approach, after demodulation, a Baseband Analyzer then detects the video modulation.

## 2.0 **SCOPE**

- 2.1** This document describes a test procedure for the laboratory and production measurement of Amplitude Modulation Cross Modulation (or AM-XMOD) that is present in Broadband Systems which carry Frequency Division Multiplexed (FDM), amplitude modulated, analog video channels.
- 2.2** A measurement procedure for AM-XMOD will be given.
- 2.3** The measurement procedure for PM-XMOD will NOT be given.
- 2.4** A more in-depth theoretical discussion of cross modulation, including a description of the physical generation and manifestations, is given in Appendix A. An outline of a derivation for an expression of PM-XMOD is given therein.
- 2.5** The baseband detection of the AM-XMOD component, and the differences between detection methods, is handled in Appendix B.
- 2.6** The PM-XMOD component's effect on the measurement of AM-XMOD, including the resulting errors in measurements of the magnitude of the detected amplitude modulation, is explained in Appendix B.

## 3.0 **DEFINITIONS**

- 3.1** Amplitude Modulation (AM) is defined as “a low frequency (i.e. baseband) signal, which is multiplied with a high frequency (i.e. carrier) sinusoidal signal, in order to modulate the peak amplitude levels of the carrier signal by 100% or less. For analog video Cable Telecommunications systems, the modulation characteristic is described as ‘downward only modulation’.” The signal has the form:

$$F_{AM}(t) = V_{pk} \mu m(t) \cos(\omega_c t + \theta)$$

**Where:**

**$V_{pk}$  ≡ The peak amplitude of the carrier signal.**

**$\mu$  ≡ The index of (amplitude) modulation:  $0 \leq \mu \leq 1$**

**$m(t)$  ≡ The baseband signal whose highest frequency component is  $\omega_m$  ( $\omega_m \ll \omega_c$ ):  $1 \geq m(t) \geq 0$ , for all  $t$ .**

**$\omega_c$  ≡ The instantaneous radian frequency of the carrier signal, considered as constant.**

**$\theta$  ≡ Arbitrary phase constant of the (sinusoidal) carrier signal.**

- 3.2** Cross Modulation can be broadly defined as “a distortion, resulting from the non-linearity of a system, which causes a carrier in the system to be modulated by the various desired signals carried on the other channels in the same system.” This general definition will cover both components of Cross Modulation distortion, AM-XMOD and PM-XMOD.
- 3.3** The “Amplitude Modulation Component” of the cross modulation, or AM-XMOD, was traditionally defined in the same manner as the definition of cross modulation given above in Section 3.2. However, in this document, an additional clarification is made. “AM-XMOD is specifically defined as the ratio of the peak to peak amplitude modulation on the test carrier, caused by the modulating signals on the other carrier signals in the system, to the peak to peak amplitude modulation on a 100% modulated test carrier. It is a measure of the relative magnitude of the voltage of the undesired modulation, to that of the desired modulation, at the fundamental frequency of the modulating signal.”
- 3.4** The conditions for the test define the modulation type, as follows:  
All test carriers are to be 100%, downward only, amplitude modulated by a synchronous square-wave signal (i.e. the same baseband signal synchronously modulates all carriers in the system), possessing a modulating frequency equal to the desired video format’s horizontal sync pulse, with a duty cycle of 50%. For the NTSC video signal format, this frequency is approximately 15.75 kHz. (Note: this will be the frequency of modulation used throughout this procedure.) However, for testing of fiber optic equipment, approximately 5% of the carrier signals may remain unmodulated in order to provide a minimum signal level within the active fiber optic components. The number, channel and frequency location of these unmodulated signals shall be included as part of the test data.
- 3.5** “Downward modulation” is understood to be amplitude modulation of a carrier from a reference carrier amplitude level such that the reference carrier amplitude level is never exceeded. (In fact, this is what is intended in the definition in section 3.1 above, as “ $1 \geq m(t) \geq 0$ ”.)
- 3.6** Refer to the Glossary and Definitions section of ANSI/SCTE 96 2008 for other definitions.

#### **4.0 EQUIPMENT**

- 4.1** The general equipment required for this test is shown in Figure 1a or 1b. ANSI/SCTE 96 2008 describes and specifies all of this equipment.
- 4.2** The multi-tone signal generator for this test must have the characteristics listed below. Refer to ANSI/SCTE 96 2008 for specifications of acceptable generators.
  - 4.2.1** The capability to produce signals on all nominal visual carrier frequencies for all of the channels in the frequency band to be tested.
  - 4.2.2** The capability to set power levels individually and to adjust the total spectrum of input signals to the proper input power level for the device to be tested.
  - 4.2.3** The capability of synchronously downward amplitude modulating all channels 100%, at a modulating frequency equal to the desired video format's horizontal sync pulse, with a duty cycle of 50%. (Note, for an NTSC video signal, this frequency is approximately 15.75 kHz.)
  - 4.2.4** The capability to turn the modulation off for individual channels.
  - 4.2.5** Spurious signals generated from within the signal source itself must be at least 10 dB below the distortion levels, to be measured. Note that, if the internal AM Cross Modulation generated by the signal source is produced in a manner similar to that produced by the "Device Under Test" (or DUT), then the levels of those distortion products must be at least 20 dB below the levels produced by the DUT to insure measurement certainty.
- 4.3** An RF Receiver is required for this measurement. Several different instruments may be used, but regardless of the specific form of the instrument the RF Receiver must have the characteristics listed below. Note: Most Spectrum Analyzers, with the appropriate Resolution Bandwidth Filter setting, can be used for this application. Refer to ANSI/SCTE 96 2008 for specifications of acceptable RF Receivers.
  - 4.3.1** The capability of tuning to the (video) carrier frequency of measurement.
  - 4.3.2** The capability of rejecting energy from image frequencies associated with the (internal) down-converting active devices of the instrument.

- 4.3.3** The capability of providing a “bandwidth selectable” Band Pass IF Filter (i.e. a ‘Resolution Bandwidth’ filter) at the input to the receiver’s RF detector. The filter is to provide selectivity to the modulated video carrier of interest and it must therefore provide adequate rejection of adjacent carrier signal energy. However, the filter must also have a bandwidth large enough to pass the modulation signal under test, so that the amplitude modulation sideband energy is properly passed through to the detector. (Namely, no significant energy of the AM sideband is to be rejected, and the conversion of phase modulation energy to AM energy, due to slope detection, is to be prevented.) Typically this is a RBW = 1 MHz.
- 4.3.4** The capability of detecting modulation via “Linear Amplitude”, or “Linear Envelope” detection, for the measurement of AM-XMOD. (Specifically, the detector chosen must be capable of providing an output voltage that is directly proportional to the magnitude of the envelope of the carrier signal.) Alternatively, a “Square Law”, or “True RMS” detector can be used. Note however, that a correction factor must then be applied, in accordance with the resulting difference in magnitude between the methods. This is a linear factor of “2”, which would result in an additional 6 dB level, assuming the units of display utilized by the Baseband Analyzer are given in ‘20\*LOG(measured voltage)’.
- 4.3.5** The capability of providing an output voltage for the baseband signal, that is directly proportional to the magnitude of the detected signal.
- 4.4** A "Baseband Analyzer" is also required for this measurement. Several different instruments may be used, but regardless of the specific form of the instrument, the Baseband Analyzer must have the characteristics listed below. Refer to ANSI/SCTE 96 2008 for specifications of acceptable Baseband Analyzers.
- 4.4.1** The capability to measure the signal level at a frequency of the modulation (which for NTSC video is 15.75 kHz).
- 4.4.2** The capability of providing a peak voltage measurement, for the measurement of AM-XMOD.
- 4.4.3** The capability of providing a “Resolution Bandwidth” Filter, either at the input to the detector or incorporated into the method of detection, in order to provide a measurement dynamic range at least 10 dB better than the AM-XMOD distortion levels to be measured. The Resolution Bandwidth must be operated with a  $BW \leq 100$  Hz. [Note: Narrower resolution bandwidths can be used and allow better measurement repeatability, but at the expense of longer measurement time.]



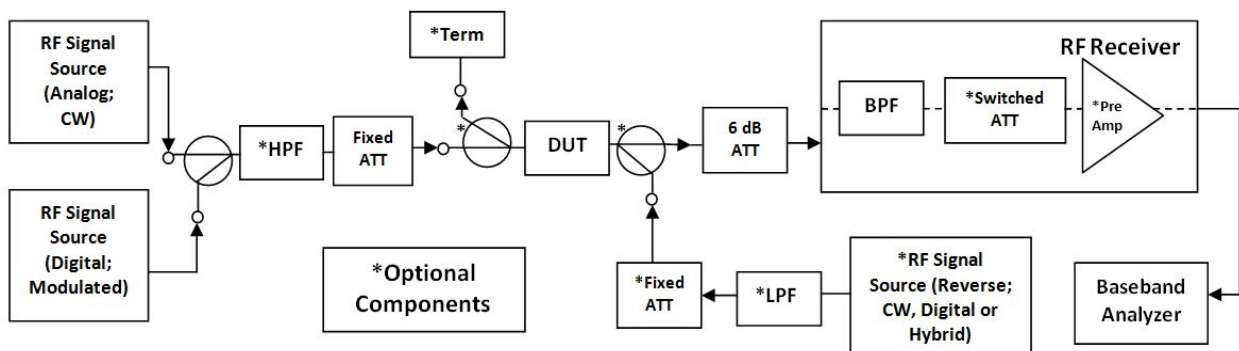
**4.4.4** The use of a separate RF Spectrum Analyzer for this application may be unsatisfactory, even if it is specified to operate at the desired frequency of 15 kHz. Typical low frequency Baseband (Spectrum) Analyzers using ‘Fast Fourier Transform’ signal processing have a noise level at least 50 dB lower than most RF Spectrum Analyzers.

**4.5** The band pass filters (BPF) for this test must meet the specifications listed in ANSI/SCTE 96 2008.

**5.0 SET-UP**

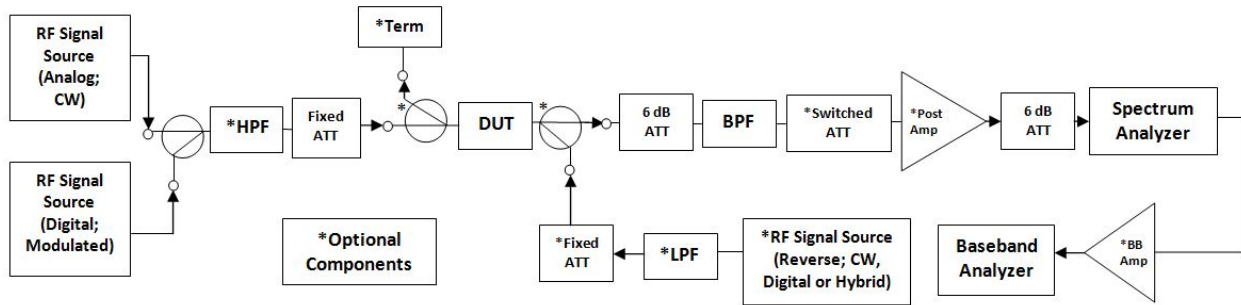
**5.1** Follow all calibration requirements recommended by the manufacturers of the Signal Generator, RF Receiver and Baseband Analyzer equipment, including adequate warm-up and stabilization time.

**5.2** Connect the test equipment as shown in Figures 1a and 1b. Please note the reverse path components marked as optional are used only with two way equipment.



**Figure 1a**

Or,



**Figure 1b**

- 5.3 If appropriate, power the Device Under Test (DUT) in a manner consistent with its use. Note that the DUT may consist of a single device or a group of devices connected together as a system.
- 5.4 Connect the video output of the RF Receiver to the input of the Baseband Analyzer.
- 5.5 Set the signal generator(s) to provide all of the signals needed for the test, as defined by the applicable frequency plan.
- 5.6 Set the appropriate signal level for each carrier, using the techniques described in ANSI/SCTE 96 2008.

## 6.0 PROCEDURE

- 6.1 Adjust and/or tune the BPF so that its pass-band response is centered on the carrier to be measured.

- 6.2** Set the RF Receiver to the settings indicated in Table 1, below. Note: If an RF Spectrum Analyzer is used as a tuned receiver, then the Span should be set to ZERO Hertz, the vertical scale should be set to LINEAR and the Reference Level adjusted until the display is near full scale.

Center Frequency	Carrier Frequency under test
Detector	LINEAR (or RMS)
Resolution Bandwidth	1 MHz
Video Bandwidth (if applicable)	$\geq 1$ MHz
Input Attenuation	$\geq 10$ dB*
Sweep Time	Auto**
Vertical Scale	Linear***

**Table 1**

- \* The RF Receiver input attenuation may be manually set to 0 dB to improve the dynamic range of the measurement, but only when one can be certain that the equipment will not be overdriven.
- \*\* The “Sweep Time” used should depend upon the instrument’s limits of calibrated operation, as well as the minimum acceptable measurement speed desired by the user.
- \*\*\* See comments in paragraph 6.2 above.

- 6.3** Turn the synchronous 15.75 kHz square-wave, 100% downward AM modulation ON for all channels.
- 6.4** Tune the Baseband Analyzer to the modulating frequency, 15.75 kHz. Ensure that the signal displayed on the Baseband Analyzer is at the full scale of the display. Ensure that the Resolution Bandwidth of the Baseband Analyzer is  $\leq 100$  Hz. (Note that a narrower baseband measurement bandwidth will produce a lower measurement noise floor and thus a more stable, more repeatable measurement.)
- 6.5** Record the peak level of the 15.75 kHz signal measured on the Baseband Analyzer as *Reference Sideband Level*.

- 6.6** Turn the modulation OFF for the carrier under test.
- 6.7** Record the level of the 15.75 kHz signal measured by the Baseband Analyzer as the **RAW AM-XMOD Level**.
- 6.8** Tune the Baseband Analyzer slightly off the 15.75 kHz signal, to a flat portion of the spectrum, within the RF Receiver’s video bandwidth.
- 6.9** Record the level measured by the Baseband Analyzer as **Noise Floor Level**. Compute the following:

$$\text{Noise Floor Delta} = \text{RAW AM-XMOD Level} - \text{Noise Floor Level}.$$

If the *Noise Floor Delta* is less than 2 dB, the bandwidth of the Baseband Analyzer can be reduced, or an appropriate post-amplifier can be added to the system. The measurement should then be made again. If, however, the *Noise Floor Delta* remains less than 2 dB, refer to ANSI/SCTE 96 2008 for the proper **Noise Floor Correction Factor**.

- 6.10** If the *Noise Floor Delta* is greater than 2 dB, the following **Noise Floor Correction Factor** should be calculated:

$$\text{Noise Floor Correction Factor: } 10 * \left| \log \left( 1 - 10^{-\left( \frac{\text{Noise Floor Delta}}{10} \right)} \right) \right|$$

A table of values calculated from this equation is presented in ANSI/SCTE 96 2008.

- 6.11** Compute the “corrected AM Cross Modulation” as:

$$\text{Corrected AM Cross Modulation, referenced to sideband} =$$

$$\text{Reference Sideband Level} - \text{RAW AM-XMOD Level} + \text{Noise Floor Correction Factor} + (6 \text{ dB, if using an RMS detector})$$

Note that this is a positive number, expressed in ‘-dB’ (i.e. negative dB), in accordance with the definition given in section 3.3. (If necessary, refer to the Glossary and Definitions section of ANSI/SCTE 96 2008 for a discussion of these units.)

## 7.0 **APPENDIX A: A DERIVATION OF CROSS MODULATION.**

In this appendix the derivations of the classical AM-XMOD expression and an expanded cross modulation expression, that embodies both an AM-XMOD and a PM-XMOD component, will be outlined separately. A thorough discussion of the consequences of each of the expressions obtained in the analysis will be presented. Most importantly, the consequences in the practical measurement of the AM-XMOD level will be detailed for each case.

As is well known, active devices used as building blocks for amplifiers are composed of semiconductor junctions. These junctions control the transport of charge (i.e. current) through the device and provide the fundamental mechanism for power gain. The nonlinear (mathematical) models for these junctions and their effects on the signals being amplified are often derived for two important cases: the ‘static’ case and the ‘dynamic’ case. It is these two cases that give rise to the two components of cross modulation.

Specifically, a simplified expression for the AM-XMOD is derived from consideration of the static case. Consideration of the dynamic case results in a more complex derivation of cross modulation, which gives rise to a more complex expression for AM-XMOD. Therefore, there are differences in the expression for the AM-XMOD in the dynamic case, from the static case. This difference conveys possible measurable effects in the level of the AM-XMOD component and can determine the ultimate repeatability of the measurement.

The more complex derivation of the general expression for cross modulation, from consideration of the dynamic case, leads to the added result that an additional component manifests itself in the form of Phase Modulation, upon the carrier signal, even though that particular’s carrier is unmodulated. Although this effect is on the carrier phase, the form is similar to the (more complex) expression for the AM-XMOD component and results in additional spectral sideband energy. This additional energy leads to consequences that can change the measured level of the AM-XMOD component, under certain conditions. Simply stated, “the existence of PM-XMOD can lead to a change in the measured level of the AM-XMOD.” This important theoretical result will also be presented in this appendix.

The treatment of the two cases (i.e. the ‘static’ and the ‘dynamic’) and the perceived effects, are based upon different assumptions about how the device behaves physically. It is these assumptions that are important in determining which case is pertinent in the practical application of the measurement.

**AM-XMOD: Derivation of Cross Modulation from the Static Case:**

If we consider a device operating in the region where its junction characteristics are based upon the ‘classic diode equation’ (e.g. a semiconductor junction whose voltage and current relationship is exponential), then modeling of the nonlinear characteristics of the device can be simplified. The nonlinear current-voltage relationship can be modeled by a polynomial function, simply by using the mathematical series expansion for the exponential function. In summary, if the signal source voltage across the junction is “not too large”, the gain compression and the generation of distortion of the device can then be modeled by a simple polynomial expression.

Specifically, the output of the device is a polynomial expansion of the power gain curve (about its DC operating point) and has the input signal as the independent variable. In mathematical form this is expressed as:

$$y(t) = k_1 x(t) + k_2 [x(t)]^2 + k_3 [x(t)]^3 + \dots + k_i [x(t)]^i \quad \{A.1\}$$

where,

$x(t)$   $\equiv$  The time dependent input waveform.

$y(t)$   $\equiv$  The time dependent output waveform.

$k_i$   $\equiv$  Real constants, representing the ‘gain coefficients’.

This expression has an important assumption associated with it. The assumption is that **‘although the output waveform is not linearly dependent on the input waveform, the output waveform is dependent only upon the value of the input waveform at the particular point in time under consideration’**. The output waveform therefore, does not depend on past values of the input waveform. In this sense the system is “static”; or, in a more mathematically correct term, it is ‘memory-less’. Consequently, a delay in the input waveform will result in a corresponding delay in the output waveform. As a result, the phase relationship between the input and output waveforms is *constant* - even though the output waveform is a distorted representation of the input. (This system is deterministic and causal.) **The consequence of the above assumption is that the junction capacitance is completely ignored. Therefore the time dependent voltage and current relationships of the junction capacitance are not included in this physical model.**

In order to derive an expression for AM-XMOD a few other assumptions are made.

First, we expect the device to exhibit a ‘weakly non-linear’ characteristic, meaning that the input signal levels are small enough as to not cause a measurably large change in the average levels of the voltages and currents of the device. (This is what was previously meant by, “the signal source voltage(s) across the junction are ‘not too large’.”) In this case, only the first three (3) terms of the expansion are considered significant, thus simplifying our final expression for cross modulation. The only terms considered then, are those associated with the linear gain (i.e.  $k_1$ ) and those associated with the slopes of the second and third order transfer characteristics (i.e.  $k_2$  and  $k_3$ , respectively). **Note that these terms do not contain any time dependencies. Therefore, they do not impart any phase dependencies onto the result.**

Second, we consider the input signal as comprised of a finite sum of equal amplitude sinusoids (i.e. we will consider a system that is of the Frequency Division Multiplexed (FDM) type, as typically found in analog cable telecommunications systems). This is expressed as:

$$x(t) = \sum_n A_n \cos(\omega_{C_n} t + \theta_{C_n}) \quad \{A.2\}$$

where,

$\omega_{C_n} \equiv$  the individual radian frequencies of the channel carriers.

$\theta_{C_n} \equiv$  the individual phases of the channel carriers (not necessarily equal).

Third, we assume that all channels have the same (peak) amplitudes. Also, that if AM modulation is present, it is the same on all channels. Therefore,  $A_n$  are given by an amplitude modulation expression,  $A(t)$ , similar to the one presented in section 3.1.

The derivation and parts of the following example are readily found in the literature and will not be presented in full detail here, but the results will be used.

We apply the assumptions given and mathematically expand (algebraically and trigonometrically) the output waveform in terms of the input waveform. Upon collecting terms we find that we have two specific categories of distortions in addition to the linear gain of the input waveform and a shift in the DC bias level of the device. The two categories of distortions are 1) those terms associated with the generation of ‘new frequencies’ and 2) those terms associated with Gain Compression. The terms associated with the generation of ‘new frequencies’ are made up of the second and third order harmonic frequencies and the second and third order inter-modulation (IMDs) frequencies, including the third order products commonly known as “Triple Beat Distortion”. Those terms associated with Gain Compression are ‘third order’ in nature; that is, the magnitudes of these terms are proportional to the slope of the third order transfer characteristic  $k_3$ .<sup>‡</sup> It must be noted that **compression** refers to a **decrease** in the signal level, at the channel of interest. This will only be true if the coefficient “ $k_3$ ” is **negative**, as

<sup>‡</sup> To be strictly correct, terms associated with Gain Compression are ‘odd order’ in nature - that is, the magnitudes of these terms are proportional to the slopes of the fifth, seventh (etc.) order transfer characteristics,  $k_5$ ,  $k_7$ , etc., as well as the third. However, for the purposes of this discussion, the higher order terms are considered inconsequential.

is the case for the physical systems we are considering.<sup>‡</sup> Therefore, **“compression terms” are distortion terms that specifically affect the amplitudes of the corresponding channels, in the output waveform.**

Gain Compression can also be broken down further, into two more ‘sub-types’ of distortion: those associated with ‘Self-Compression’ and those associated with ‘Cross-Compression’. The two sub-types do show an important difference in their terms. The magnitudes of the Self-Compression terms depend on the input amplitude level solely at the carrier frequency (i.e. channel) of interest - hence the name given to the sub-type. The magnitudes of the Cross-Compression terms depend however, not only on the input amplitude level at the channel frequency of interest, but also on the ‘sum-of-the-squares’ of input amplitude levels of all the other channel frequencies in the system. As a result of these two facts, important characteristics exist between the two sub-types of Gain Compression. The most obvious characteristic is a consequence of these two sub-types being caused by the same physical occurrence. Being that, **when the amplitude of the channel of interest is ‘Zero’ (i.e. when the channel of interest is turned OFF, in the system) then distortion due to the Gain Compression, whether Cross-Compression or Self-Compression, ceases to exist for that particular channel.** In fact, the part of the Cross-Compression energy (i.e. Cross-Compression distortion term) of that particular channel, that exists at all the other channels, disappears and alleviates some of the distortion on those channels. Another important characteristic, which is the important difference between the two sub-types, is that in a physical system, Self-Compression cannot be readily separated from the channel’s carrier amplitude level and therefore cannot be measured practically, while Cross-Compression levels can be measured in a practical way.

Note that, our focus is on the mathematical terms associated with “Cross-Compression”. These terms then give an output expression, for any particular channel of interest, which includes the AM-XMOD component.

Consider a device (or system) of the type that can be described by equation {A.1} and has, as its input, a signal described by equation {A.2}, including amplitude modulation of the type given in section 3.1. If we include the previously stated simplifications, an output expression for a particular channel can then be written. If we exclude all second order terms, as well as all harmonic and inter-modulation terms, the expression is:

$$y_o(t) = k_1 A_1(t) \{ 1 + (3/2) (k_3 / k_1) \{ (1/2) [A_1(t)]^2 + [A_2(t)]^2 + [A_3(t)]^2 + [A_4(t)]^2 + \dots + [A_n(t)]^2 \} \} \cos(\omega_c t + \theta_c) \quad \{A.4\}$$

where, ‘n’ is the number of channels present in the system (n ≥ 2 is implied).

Note the ‘Self-Compression’ term, given by the expression “[A<sub>1</sub>(t)]<sup>2</sup>”. The other squared terms represent the individual ‘Cross-Compression’ components. An expression for the amplitude of

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<sup>‡</sup> Alternatively, it can be stated that the components of compression are always produced with a 180° (i.e. π radians) phase, in reference to the ‘original’ (or, linearly amplified) channel of interest.



the particular channel, which includes the application of all simplifying assumptions (including that of synchronous modulation), is then given by:

$$y_o(t) = k_1 A_1(t) \{ 1 + (3/2) (k_3 / k_1) [V_{pk}]^2 (n^{-1/2}) [\mu m(w_{MOD}, \phi_{MOD}, t)]^2 \} \cos(w_c t + \theta_c) \quad \{A.5\}$$

The expression given in {A.5} contains simple amplitude modulation, hence the description as “AM-XMOD”. A useful expression for the value of the AM-XMOD can then be given as (for  $n \gg 1$ ):<sup>†</sup>

$$\text{AM-XMOD\% (static case)} \equiv (3/2) (k_3 / k_1) (n) [V_{pk}]^2 (100) \quad \{A.6\}$$

From the above expressions, it is apparent that any channel that is amplitude modulated will transfer some of that energy of modulation to all the other channels present in the system, as previously stated. It follows from this derivation that, if all channels in the system are synchronously modulated, then the total contribution of the AM-XMOD distortion at a particular channel should be the exact sum of the AM-XMOD distortions contributed by each individual channel. (This is represented by the ‘(n)’ term in {A.6}.)

An important consequence of the above is that, any system that doubles the number of channels, would then double the level of the AM-XMOD distortion. Equivalently, a doubling of the number of channels in the system increases the AM-XMOD level by 6 dB. Another consequence is that, if the number of devices in a system doubles, then the AM-XMOD distortion level of the system will also double; once again, this represents a 6 dB increase in the AM-XMOD energy at any particular channel of interest. (Note: this last statement is true only if the input levels to each device in the system remain unchanged.)

If one considers the amplitude levels of the channels, then it is observed that for each 1 dB change in amplitude level for all the channels in the system, the AM-XMOD distortion level for a particular channel will change by 3 dB and the AM-XMOD distortion ratio changes by 2 dB. Equivalently, for each doubling in level of all the channels in the system, the AM-XMOD distortion level will quadruple, for each and every channel. (Note: This is represented by the ‘ $[V_{pk}]^2$ ’ term in {A.6}.) This implies a fixed 2:1 relationship, in dB, of the ratio of the observed third order distortion level and the amplitude of all the channels. In fact, a fixed relationship does exist between the two types of third order distortion; namely, AM-XMOD (as described in the above derivation) and “Triple Beat” distortion. It is an important and useful fact that indeed a relationship does exist.

Finally, the only way to measure the energy of the cross-compression term at a particular channel is to disable the modulation of that particular channel. (That is: if one is to measure the energy of the AM-XMOD distortion at a particular channel, then one needs to turn OFF the modulation on that channel.)

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<sup>†</sup> A complete derivation is given in the references.

One last issue, which has caused much misinterpretation and debate, regards the actual amplitude reference level that should be used for the measurement of AM-XMOD. The controversy resides in whether or not the AM-XMOD energy is measured referenced to the amplitude of the channel carrier when the modulation for that particular channel is OFF, or ON, or even whether the reference amplitude should actually be the 15.75 kHz sideband of the channel carrier's modulation. Since, for the fixed type of modulation signal as given in this document there exists a fixed relationship between these quantities, one needs only to make clear exactly which reference was used in the measurement.

### **AM-XMOD and PM-XMOD: Derivation of Cross Modulation from the Dynamic Case:**

As stated in the introduction and as formulated in the previous section, AM-XMOD arises from the static, nonlinear nature of semiconductor junction devices. In particular, an expression for AM-XMOD was derived from the exponential current-voltage relationship, which typically exists in a bipolar junction diode or transistor. However, practical devices exhibit dynamic physical phenomena that change the characteristics of the non-linearity. **If we consider the dynamic characteristic of the semiconductor junction, we must include the time dependent current-voltage relationship of the junction capacitance(s) of the device(s).** One now asks, "What is the effect of including the time-dependent nature of the junction?" The answer to this question is, "that we need to add to our previous model the capacitance of the junctions, which represent the physical non-linear time dependent nature of the transport of charge".

In general, the solution to the problem of the behavior of the nonlinear junction then becomes one of using advanced mathematical techniques in order to solve the resultant nonlinear differential equation. The solution is not algebraic – the solution depends, not only on the magnitude of the source signal, but also on the time-rate-of-change of the source signal due to the (non-linear) junction capacitance.

A fundamental point can be ascertained from this 'second order' nonlinear model. That is, both the Collector-Emitter and the Collector-Base junctions impart a time dependency onto the solution. The importance of this phenomenon is far from academic. It is the physical reason for the existence of PM-XMOD (i.e. AM to PM conversion). In short, the simple existence of nonlinear junction capacitance in a semiconductor device (used to amplify any number of amplitude modulated channels), will impart an added phase distortion to the channel signals themselves.

In general, because the magnitude of the junction capacitance depends upon the magnitude of the voltage across it, any time dependence in the signals being processed by the junction is then translated as a time dependent capacitance. Moreover, the internal structure of the device includes an inherent 'feedback capacitance'. The transfer function of the device (i.e. the input to output voltage relationship) no longer is a simple non-linear curve whose characteristic is independent of time (or of frequency). This relationship must now be described by a nonlinear differential equation whose coefficients are not simple (i.e. constant). In fact, the coefficients are

nonlinear themselves, having a dependency upon the dependent variable (e.g. the output signal). In conclusion, time dependent (amplitude and phase) distortion can result.

Clearly, the expression for the gain compression cannot be modeled by a polynomial function with constant, time independent coefficients. To solve this general problem, one can use a Volterra series expansion. The Volterra series expansion is an approximation to the general solution. It consists of a series of terms, which are an approximating expansion to the nonlinear (integral-differential) system equation. Each term therefore, is a convolution integral of the input waveform and an approximating system sub-function. Each system sub-function has a complexity that is the order of the non-linearity considered. Hence, the first expansion term in the series (i.e. the first convolution integral) represents the solution to the linear system, while the third expansion term represents the solution to the third order non-linearity of the system. In summary, the Volterra series expansion is a sum of terms, wherein each term consists of a convolution integral that represents the system solution to a particular order of (non-) linearity.

In addition, each system function (contained within the convolution integral) is an independent (i.e. orthogonal) impulse response, whose arguments contain the fundamental time constants of the system. An input waveform composed of many (discrete) frequencies will then cause multiple impulse responses by the non-linear system. In essence, each Volterra term's solution (i.e. result) partly depends upon the particular set of (discrete) frequencies of the input waveform considered. The Volterra series allows a system, including one with 'memory', to be regarded as a combination of systems, wherein one is a linear system and the others are nonlinear systems of order two and higher. In fact, the solution given by the power series of the static case should resemble the dynamic case, since the power series are a simplification of the more general solution obtained in the dynamic case, using the Volterra series.

Using the Volterra theory, we can represent our nonlinear system as:

$$y(t) \equiv y_1(t) + y_2(t) + y_3(t) + \dots + y_i(t) \quad \text{higher order nonlinear systems \{A.8.a\}}$$

In general,

$$y_i(t) \equiv i^{\text{th}} \text{ order nonlinear subsystem of the Volterra series expansion,}$$

$$\text{where} \quad i > 1$$

Concentrating on the 1<sup>st</sup> and 3<sup>rd</sup> orders and assuming our input is  $x(t)$ , we have:

$$y_1(t) = \int h_1(\tau) x(t-\tau) d\tau \quad \text{and} \quad \{\text{A.8.b}\}$$

$$y_3(t) = \iiint h_3(\tau_1, \tau_2, \tau_3) x(t-\tau_1) x(t-\tau_2) x(t-\tau_3) d\tau_1 d\tau_2 d\tau_3 \quad \{\text{A.8.c}\}$$

Since each term in the Volterra series is a convolution integral of a system function and the input waveform, the solution of the output response can be determined with the methods of 'Fourier Analysis Theory'.

Using the ‘Fourier Theory of Transformation’ for the system functions gives, in general:

$$H_i(jw_1, jw_2, \dots, jw_i) = F\{ h_i(\tau_1, \tau_2, \dots, \tau_i) \} \quad \{A.9\}$$

$H_i$  can be regarded as the transfer function of the  $i^{\text{th}}$  order nonlinear subsystem, whose arguments depend upon a particular set of discrete frequencies of the input waveform, as well as the time constants of the system.

Whether it is from measured data, or from a computational system analysis, the dependence of the system response at these frequencies must be known. The implication of this is not subtle; some basic knowledge of the system's response to the input excitation is mandatory. Obviously, knowledge of the input signal is ‘a priori’ for a solution.

A derivation of the dynamic case, using the Volterra series, will not be given since it can be found in the literature. However, the assumptions used in the derivation will be stated and the results will be used to define the PM-XMOD component. An expanded expression for the AM-XMOD component will be given, as well. The differences between the static case and the dynamic case for the AM-XMOD component can then be related to actual changes in the signal characteristics of the measured distortion.

The first assumption refers to the stability of the active device(s) in the particular amplifier circuit of interest. The use of the Volterra series requires the device to be (asymptotically) stable, so that the physical characteristics of the device do not change.

The second assumption is that the non-linearity of the device is ‘incremental’, meaning that the distortions are a fraction of the energy (i.e. < 40 dB) below the channel carrier amplitudes. (This purposely is not precise; it only illustrates the magnitude of ‘incremental’.) The output response will then be considered adequately described by the first three terms of the Volterra expansion (i.e. only the second and third order non-linearities, in addition to the linear response, will be considered).

The third assumption is that the input waveform should not produce very large peak signal levels. (Again this is not precise, but is important to understand that the formulated result depends upon the convergence of the series expansion.) Note that this third assumption is closely related to the second assumption, but is used for a different purpose. It will allow us to formulate a ‘closed-form’ solution without specific details of all of the system’s transfer functions.

As described previously, the higher order transfer functions of the non-linear system (with memory) are dependent on more than one frequency component. We will greatly simplify the problem by considering these transfer functions to be ‘single-frequency dependent’, which is our fourth assumption. Computationally, this means that the multivariate (i.e. multi-dimensional) transfer functions described by equation {A.8} are then considered to be ‘single-variable’ functions, where the variable is the sum of the input frequency components, instead of being dependent upon each and every individual frequency component. The physical implication is, that all the distortion products being of the same type and being composed of input signals that

have the same amplitude levels, would then have the same distortion levels for the same distortion frequency, regardless of the frequencies of the comprising (input) signals. (Or, any pair of third-order distortion products that occur at the same frequency will have the same magnitude of distortion no matter what two, or three, different fundamental frequencies of the input signal they are comprised from.) Note that, distortion products of the same type, located at different (sum and difference) frequencies, could have different levels. The terms of equation {A.8} would then involve only one-dimensional convolutions. In addition, the non-linear subsystems of the equation can be regarded as composed of ‘zero-memory’ networks, while the linear term would then be ‘time-invariant’. (This would be the case if the junction capacitance were constant.)

Each term in the output response then, is a form of the input response modified by the magnitude and phase of the particular transfer function. As in the case of frequency dependent linear systems, the great advantage of single-frequency dependent non-linear systems is, the second and the third order harmonic levels alone can characterize them.

Typically, the linear gain of a wide-band device is flat, within the band of interest. Many wide-band devices will also exhibit transfer characteristics that have measurable third order distortion levels that are approximately equal, when the sum of frequencies of the comprising signals is approximately equal. This is especially true of wide-band devices whose operation is closely approximated by ‘unilateral’ operation, or the ‘feedback’ of the device has no appreciable (close-loop) phase. Applying these assumptions allows a formulation for the problem at hand. A useful expression can be derived for the cross modulation term of a time dependent non-linear system, when the input is a sum of amplitude modulated sinusoids.

The fundamental output at a particular channel is given by:

$$y_{\text{fund}}(t) = k_1(w_c) V_{\text{pk}} \cos(w_c t + \theta_c - \phi_1(w_c)) \quad \{\text{A.11}\}$$

The total output of the Cross Modulation term at the channel carrier (denoted by ‘ $w_c$ ’), due to the sum of the cross modulation contributions from all the other modulated channel carriers (denoted by ‘ $w_{nc}$ ’), is given by:

$$y_{\text{xmod}}(t) = (3/4) [V_{\text{pk}}]^3 k_3(w_i) \cos(w_{\text{mod}} t) \{ \cos(w_c t + \theta_c) [\sum_n \cos(\phi_3(w_{nc}))] + \\ + \sin(w_c t + \theta_c) [\sum_n \sin(\phi_3(w_{nc}))] \} \quad \{\text{A.12}\}$$

where:

$$w_i \equiv w_c \pm w_{\text{mod}}, \text{ the frequency of the sideband} \quad \{\text{A.13}\}$$

Notice that, the added relationship of the third order phase exists as a function of frequency at the channel carriers. Therefore the cross modulation signal is composed of two signals, each in quadrature with the other, causing the effect of amplitude and phase modulation. This modulation is, in effect, ordinary QAM, where the ‘in-phase’ signal (I) and the ‘quadrature’

signal (Q) are combined so to give a signal which is both amplitude and phase modulated; and in such a manner as to be able to resolve the I and Q components separately, when demodulated.

Combining both components  $\{y_{fund}(t)$  and  $y_{xmod}(t)\}$  and a simplification of terms, gives an expression for the total output at the particular carrier of interest:

$$y_c(t) = k_1(w_c) V_{pk} [1 + b \cos(\varphi) \cos(w_{mod}t)] \cos[w_c t + \theta_c - \phi_1 + b \sin(\varphi) \cos(w_{mod}t)] \quad \{A.14\}$$

where:

$$b = (3/4) [V_{pk}]^2 \sqrt{\varepsilon} [k_3(w_i) / k_1(w_c)] \quad \{A.15\}$$

$$\varphi = -\psi + \phi_1 \quad \{A.16\}$$

$$\psi = \tan^{-1} [\sum_n \sin(\phi_3(w_{nc})) / \sum_n \cos(\phi_3(w_{nc}))] \quad \{A.17\}$$

$$\varepsilon = n + 2 \sum_{q=1 \text{ to } (n-1)} \sum_{p=(1+q) \text{ to } n} \cos[\phi_3(w_{qc}) - \phi_3(w_{pc})] \quad \{A.18\}$$

The final form shows that the output at a particular carrier of interest can be expressed as:

$$y_c(t) = k_1(w_c) V_{pk} [1 + \text{AM-XMOD} \cos(w_{mod}t)] \cos[w_c t + \theta_c - \phi_1 + \text{PM-XMOD} \cos(w_{mod}t)]$$

where:

$$\text{AM-XMOD (dynamic case)} \equiv b \cos(\varphi) \quad \{A.19\}$$

$$\text{PM-XMOD} \equiv b \sin(\varphi) \quad \{A.20\}$$

These expressions show an example of how the phase of the third order distortion can affect the magnitude of the AM-XMOD. They also demonstrate the creation of phase modulation from amplitude modulation, in amplifiers.

Close examination of the expressions given in {A.14} through {A.18} reveal important characteristics of the resultant cross modulation signal. The first characteristic regards the frequency dependence of the AM-XMOD and PM-XMOD expressions. Note that if we consider the system to have no frequency dependence, then:

$$\varepsilon \Rightarrow (2n)^2$$

$$\psi \Rightarrow 0$$

$$\varphi \Rightarrow 0$$

$$\text{so that } b \Rightarrow (3/4) [V_{pk}]^2 (2n) [k_3 / k_1]$$

therefore,      AM-XMOD (dynamic case)  $\Rightarrow$  AM-XMOD (static case)  
 and              PM-XMOD  $\Rightarrow 0$

which is the expected result.

In general, any particular third order distortion product will have a given phase component, not necessary equal to any other third order phase component, and therefore the total cross modulation distortion will neither be completely of the AM type or of the PM type. In general then, the resultant cross modulation distortion will always be a compilation of the two types. In practice, however, the higher frequencies tend to exhibit a larger PM-XMOD component, especially at the frequencies where the bandwidth of the device starts to ‘roll-off’. The magnitude of the PM-XMOD is obviously dependent upon both the frequency of the distortion and the magnitude of the junction capacitance.

The first result of the dynamic case considered is that there is, in general, no fixed relationship between AM-XMOD of a two-channel system and that of a multi-channel system. To be specific, one cannot in general measure the AM-XMOD from any pair of channels and use this result to calculate the resultant AM-XMOD due to multiple channels. The resultant multi-channel cross modulation level depends upon the relative differences of the phases of the third order distortions, and therefore do not necessarily add on a ‘power basis’. Sometimes, an effective ‘average’ is assumed and the sum is added as “15 log(x)” for practicality, but this causes confusion as to whether or not the products add on a ‘voltage’ basis (i.e. 20 log(x) ) or on a ‘power’ basis (i.e. 10 log(x) ).

The second result is that, depending upon the phase variation (vs. frequency) of the third order distortion characteristic of the device (or amplifier, or system), the inclusion, or the removal of, several synchronously modulated carriers can produce either an improvement in, or a degradation of, the measured AM-XMOD. A corollary to this is, that in the dynamic case Synchronous Modulation does not necessarily produce the worst possible measured AM-XMOD. In fact, random distribution of the phase of the modulating signal, on each carrier, could produce measurements that are worse on average. A second corollary, slightly more obscure from the first, relates the effect of the increase in energy of the AM-XMOD component, when the magnitudes of the carrier amplitudes are increased: The effect may not be closely related (in magnitude) to a similar effective increase in the number of carriers (i.e. channels) added into the system, as would be expected from the static case. This would especially be true when adding of channels occurs at the high frequency band of the system spectrum.

The third and fourth results regard the measured (i.e. effective) magnitude of the resultant cross modulation upon detection. The third result is that, detection of the AM-XMOD component in the general case is not the same it is in the static case – the addition of the time dependent amplitude in the AM-XMOD component causes much more uncertainty in the magnitude of the detected signal. Therefore, a ‘peak detected’ AM-XMOD signal would give, in general, a much different result than a ‘true square law’ (i.e. ‘true RMS’) detector.

The fourth result regards the detection of vestigial sideband signals. When a vestigial sideband filter is used as part of the detection method, then the means for conversion of PM-XMOD to AM-XMOD exists. In essence, the effective AM-XMOD magnitude that is detected changes as a function of the PM-XMOD component present.



## 8.0 APPENDIX B: THEORY OF MEASUREMENT OF AM-XMOD

In this section, the basic theory of the measurement of AM-XMOD will be reviewed. First, a description of the test signal (i.e. “the stimulus”) will be given. Second, a description of the expected response signal will also be given. The response signal is the result of the test signal being passed through a nonlinear system, like the one described in Appendix A. The signal then contains the expected response, with which we must make a measurement of distortion. Finally, a brief outline will be given of the types of measurements on the resultant distortion that can be made and the corresponding equipment used. Important to the discussion are the different detectors that can be used for the measurement and their possible affects on ‘dynamic range’, ‘accuracy’ and ‘repeatability’.

In the past, the definition of AM-XMOD has been interpreted in different ways by the industry. Some have specified the AM-XMOD as, “the ratio of the peak of the non-modulated carrier signal to the peak of the undesired modulation”, while others specify AM-XMOD as, “the ratio of the peak of the first sideband, for a 100% modulated carrier, to the peak of the undesired modulation”. The difference depended mainly upon the type of instrument available to make the measurement of the resulting distortion. As it will be shown, there is an exact relation between the two results, if the stimulus is consistent. The theoretical difference between these measurements is stated as: the AM-XMOD referenced to the peak of the first sideband of a 100% square-wave modulated carrier signal is 10 dB worse than the AM-XMOD referenced to the peak of the non-modulated RF carrier. For results reported according to this procedure, **all AM XMOD measurements and specifications should always be referenced to the peak of the first sideband of a 100% square-wave modulated carrier (i.e. all reported results for the AM-XMOD measurement should always include this factor).** The primary importance for this is ‘consistence in reporting results’.

The primary stimulus is a sinusoidal waveform, whose typical frequency is in the RF range, and whose amplitude is varied in direct proportion to a low frequency square-wave. The frequency of the square-wave is the rate of modulation. The rate of modulation is chosen to be the same rate as the Horizontal Sync Frequency of the video format chosen (in this case it is 15.75 kHz, as defined in the NTSC analog video format). The actual stimulus is then made up of a number of primary signals (i.e. more than two), whose peak carrier level, carrier frequencies and total number are exactly specified. This stimulus is so chosen as to be a definitive representation of the waveform most pertinent to the actual signals that the system (or device) transports, since most of the energy of the video waveform (i.e. the energy of the modulation) is carried in the Horizontal Sync Pulse. Therefore this is the waveform that will contribute significantly to the generation of AM-XMOD.

It is important to note that the test, as previously outlined in the procedure, is designed to indicate a certain level of distortion generated from within the system (or device), which can be measured in a repeatable manner. It is not, however, designed to be representative of actual performance even though the carrier levels may be the same, because the actual video signal does not contain modulation at a 100% level. As well there are other factors, such as the Vertical Sync Pulse and the actual video level contained within the picture information, that can affect the magnitude of the measured AM-XMOD within a given measurement bandwidth. The methodology of the test therefore minimizes the subjective performance that can be incurred by these factors and provides the basis for correlation to actual system performance.

Since the primary stimulus signal also serves as the reference for the (relative) measurement of AM-XMOD distortion, this signal should first be examined. Since, as well, the stimulus signal can be examined in both the ‘time’ domain and ‘frequency’ domain, it is important to view the signal in both domains graphically. Figure A1 shows the time response of a carrier signal without modulation. Figure A2 shows the same signal in the frequency domain, where the magnitude of the Power Spectral Density is plotted versus frequency. Both of these signals can be represented by the following (time domain) expression:

$$y_1(t) = V_{pk} \cos(\omega t + \theta) \quad \{\mathbf{B.1}\}$$

Where:  $\theta = 0$ , can be chosen for simplification.

This expression will serve as our reference, in order to simplify future discussions.

Figure A3 shows the signal when modulated with a low frequency square-wave that has a 50% duty cycle. As shown, the carrier is pulsed ON and OFF for equal periods of time, creating a symmetric, pulsed RF signal. Note that this represents 100% downward amplitude modulation of the carrier.

Since the depth of the (downward) modulation, from the peak of the envelope of the carrier, represents the analog level of the video, we can see that no video information is embodied by this test signal. In other words, the test signal carries no video energy and the envelope of the modulation corresponds to the energy of the Horizontal Sync pulse only, as desired.

The test signal, as shown, can be characterized by the following expression:

$$y_2(t) = V_{pk} \mathbf{u} \mathbf{a}(t) \cos(\omega t + \theta) \quad \{\mathbf{B.2}\}$$

Where:  $\mathbf{u} \equiv$  the index of modulation, chosen to be 100% (i.e. a magnitude of 1.0).

$\mathbf{a}(t) \equiv$  the modulating signal; a square-wave, with a duty cycle of 50% and frequency of 15.75 kHz.

An expression can be given for the term  $\mathbf{a}(t)$ , using a Fourier series:

$$\mathbf{a}(t) = \left\{ \frac{1}{2} + \frac{2}{\pi} \left[ \sum_k \frac{1}{k} \sin(k \omega_{mod} t + \phi) \right] \right\}; k = 1, 3, 5 \dots \infty \quad \{\mathbf{B.3}\}$$

Where:  $\omega_{mod} = 15.75$  kHz, the rate of modulation.

$\phi = 0$ , chosen for convenience.

Note that, given the modulation is “downward only”, the peak magnitude of the carrier due to  $a(t)$  is “ $V_{pk}$ ” when measured in the time domain, as it is when the carrier contains no modulation.

Figure A4 shows the signal ‘ $y_2(t)$ ’ in the frequency domain, where the magnitude of the Power Spectral Density is again plotted versus frequency. Comparing Figure A2 with Figure A4, we see that the amplitude of the signal at the frequency of the carrier has a magnitude that is 6.0 dB lower when modulated. We can also see from Figure A4 that the magnitude of the first sideband is located at a distance of 15.75 kHz (i.e. the “fundamental” sideband frequency) above and below the carrier frequency. It will suffice to point out that the magnitude of either of the first sidebands should be 3.9 dB below that of the modulated carrier and 9.9 dB below that of the same carrier signal when it is not modulated (as shown in Figure A2). Proper modulation of the (source) carrier should always result in energy levels of the sidebands, above and below the carrier frequency, being the same (i.e. the sideband energy is symmetric about the carrier, as measured in the frequency domain).

The previous discussions showed the exact relationship between the components of the stimulus signal and the unmodulated carrier, in the frequency domain. It also demonstrates the need to exactly specify which of three signal levels were used as the reference during the measurement of AM-XMOD; the level of the unmodulated carrier, the level of the modulated carrier, or the level of the first sideband of the modulated carrier. The factor of ~10 dB between the two interpretations of the AM-XMOD measurements, as initially described, is shown to be the difference between the levels of the first sideband of the modulated carrier and that of the unmodulated carrier itself, as measured by the Power Spectral Density.

AM-XMOD is a result of each carrier signal that is modulated, transferring a small fraction of its sideband energy to every other carrier in the system. The response signal therefore is separated into two parts: one is the reference and the other is the raw measurement of the level of AM-XMOD itself. The reference signal is the first sideband of the modulated carrier under measurement, as previously stated. The raw AM-XMOD measurement is performed on the respective carrier signal when only residual sideband energy (i.e. the AM-XMOD), which is transposed to it by the (nonlinear) system, is present. Synchronous modulation of all carriers provides a “worst case” measurement. It also provides a means of measurement of the peak voltage of the modulation that follows a “voltage law” (i.e. the total voltage of the AM-XMOD level is proportional to the number of individually modulated carriers in the system).

The basic requirements of the measurement necessitate a means of separating the carrier energy from the sideband energy, for each individually desired carrier. Therefore the resultant signal that contains the AM-XMOD distortion must be demodulated. As well, since there is usually a very large difference between the levels of the first sideband of the modulated carrier (i.e. the ‘reference signal’) and that of the residual distortion (i.e. the AM-XMOD), an extremely large dynamic range is required from the measurement equipment.

Demodulation is best accomplished at a fixed Intermediate Frequency, or “IF”. (Typical IF frequencies are 21.4 MHz, 10.7 MHz and 455 kHz.) At a fixed IF, the performance of the Demodulator is largely independent of the carrier frequency under test. However, a fixed IF requires a capability of frequency translation over a wide range of carrier frequencies.

A typical high performance test instrument used to accomplish the above tasks is the ‘Spectrum Analyzer’ (or, ‘SA’). Another instrument that can be used is a ‘Receiver’. Although there are Receivers classified as “general purpose” or “special purpose”, the SA can be classified as either, depending upon the specific use. However, there are usually three differences that distinguish an instrument as either a ‘Receiver’ or as a ‘Spectrum Analyzer’. The first two differences concern how the two instruments accomplish their frequency translation and how they display the energy they measure. The SA normally contains a “swept local oscillator” (or “swept LO”) and a wide bandwidth circuit for translation of the waveform energy. This allows the instrument to efficiently display the spectral energy of a waveform over a wide frequency range. The Receiver, however, usually contains a “discrete local oscillator” and a relatively selective circuit for frequency translation of the waveform energy. This allows the Receiver to efficiently display the power of a waveform within a narrow frequency range, at any number of specific (i.e. desired) measurement frequencies (e.g. the frequency of the measurement can be abruptly changed and the power reading taken immediately). The third difference concerns how the two instruments accomplish their task of demodulation. The SA normally contains only a “peak sampling detector” for demodulation and detection of the waveform’s energy. The Receiver, however, normally contains a variety of different detectors, in order to optimize the dynamic range and accuracy of the measurement, depending upon the waveform’s characteristics.

A more detailed discussion of the two instruments is covered in ANSI/SCTE 96 2008. It should be noted that there are a number of subtle consequences of the AM-XMOD measurement when one instrument is chosen over the other. Many of these involve measurement speed and repeatability and are a matter of measurement technique. However, four issues are most important to the measurement accuracy and necessitate elucidation in the present procedure.

The first of these is “contribution of distortion by the measurement equipment” and must be avoided for proper dynamic range. Since a Receiver or a SA is capable of generating distortion if ‘overloaded’, proper care must be taken to ensure this is avoided. If need be, external attenuation or pre-selection filtering must be included in the measurement setup. Commonly, rejection of energy from ‘image frequencies’ and/or ‘adjacent carriers’ is needed externally to the SA, due to its wide bandwidth characteristics.

The second issue is the “contribution of noise by the measurement equipment” and is a concern when the minimum levels of the distortion are obscured by the noise inherent in the measurement system itself. A Spectrum Analyzer generally has a relatively high Noise Factor, due to the wide bandwidth nature of its internal circuits. Many times this can be overcome by including an external pre-amplifier in the measurement setup – however, proper selection of amplifier’s performance is essential, to avoid contribution of distortion by the pre-amplifier (and/or the SA, from too high a signal level). Most receivers contain an internal pre-amplifier that can be (automatically) switched into the measurement path as needed. However, depending upon the internal pre-amplifier performance, selective filtering external to the Receiver may then need to be added, to limit overloading and generation of internal distortion.

The third issue is the choice of the ‘Resolution Bandwidth’, which is basically the IF filter bandwidth selection. Too low a value will attenuate the sideband energy of the modulated carrier. Too high a value will allow too much noise into the post-demodulation process, which could totally obscure the sideband energy, limiting the measurement dynamic range. Or, if the sideband energy were not obscured, enough noise would cause unwanted uncertainty in the measurement. A typical value of 1 MHz is usually chosen as a good compromise.

The fourth issue concerns the process of Demodulation and detection of the sideband energy. As long as proper consideration is given to the factors that affect the measurement, separation of these two processes most often yields a higher dynamic range and repeatability. Many of these factors have to do with actual ‘post-demodulation’ processing of the sideband energy, but the method of demodulation contributes to this process as well. Therefore it is important to first discuss the various means of detecting amplitude modulation.

The essence of amplitude demodulation is “linear detection of the envelope of the carrier signal amplitude”. The ideal amplitude demodulator would be one that produces a baseband waveform that is an exact replica of the (time variant) envelope of the carrier signal. One method of accomplishing this is to register the peak value of each and every positive (or negative) excursion of the (sinusoidal) carrier signal. Since both the envelopes of the positive excursions and the negative excursions of the carrier signal contain the same sideband energy (i.e. information of the modulating signal), only one of these envelope signals is required. Two general means of accomplishing this first method of demodulation would be the “Peak Detection, Sample and Hold” (or ‘PD-SH’) and the “Rectification and Averaging” (or ‘R&A’). In general, the R&A methods of demodulation provide the best dynamic range; that is, it provides the lowest response to noise. The PD-SH method can provide a dynamic range arbitrarily close to that of the R&A, as long as the (post demodulation) averaging time is appropriately increased. The PD-SH detection method is commonly used in the Spectrum Analyzer instrument. The R&A detection method is generally one of the options available in an RF Receiver instrument.

Another method of accomplishing amplitude demodulation is to directly translate the waveform of the carrier envelope (i.e. information of the modulating signal) to a baseband waveform. In the purest sense, this is done through multiplication of the carrier by itself (i.e. using synchronous detection). An alternate means of accomplishing this second method is “Square Law” (or ‘SQL’) detection. This alternate means is an asynchronous detection method and usually requires constant, precision control (i.e. ‘AGC’) of the amplitude of the (IF) signal to be demodulated. The ‘SQL’ detection method provides excellent dynamic range. In fact, close to that of the R&A method. In addition, a ‘true’ SQL detector, typically termed a ‘RMS’ detector, provides the actual “root-mean-square” value of the signal as well as for the noise. This method is also usually an option available in an RF Receiver.

Further discussion of each of these detection methods is carried out in some detail in ANSI/SCTE 96 2008. As pointed out in that document, each of these methods has different performance levels when detection of (sinusoidal) signals is required in the presence of noise. There is, however, one other issue of AM demodulation that is of importance when detecting AM in the presence of PM. It is a fact that the various methods of detection can give widely varying measurement results, if the PM is present in significant magnitude. In general, any means of (analog) ‘peak detection’ of the carrier envelope will be more susceptible to phase variations of the carrier frequency than would the method of analog multiplication. This is because ‘peak detection’ methods rely on ‘uniform sampling’ of the peak carrier amplitudes (e.g. the phase at which the carrier peak amplitude occurs, needs to be kept constant). Any time difference between the occurrences of the peaks of the carrier amplitude, result in a distortion of the envelope signal. The ‘analog multiplication’ method inherently reduces (i.e. theoretically eliminates) the phase dependence, due to its ‘synchronous nature’. Therefore, if these phase variations are of a time dependent nature, so will be the measurements. Many times these variations can be ‘smoothed out’ by the process of averaging, carried out in the post-demodulation process. Perhaps the biggest advantage of the SQL detector is that it provides the best immunity from phase effects; that is, it provides the best immunity from the effects of PM-XMOD on the accurate detection of AM-XMOD.

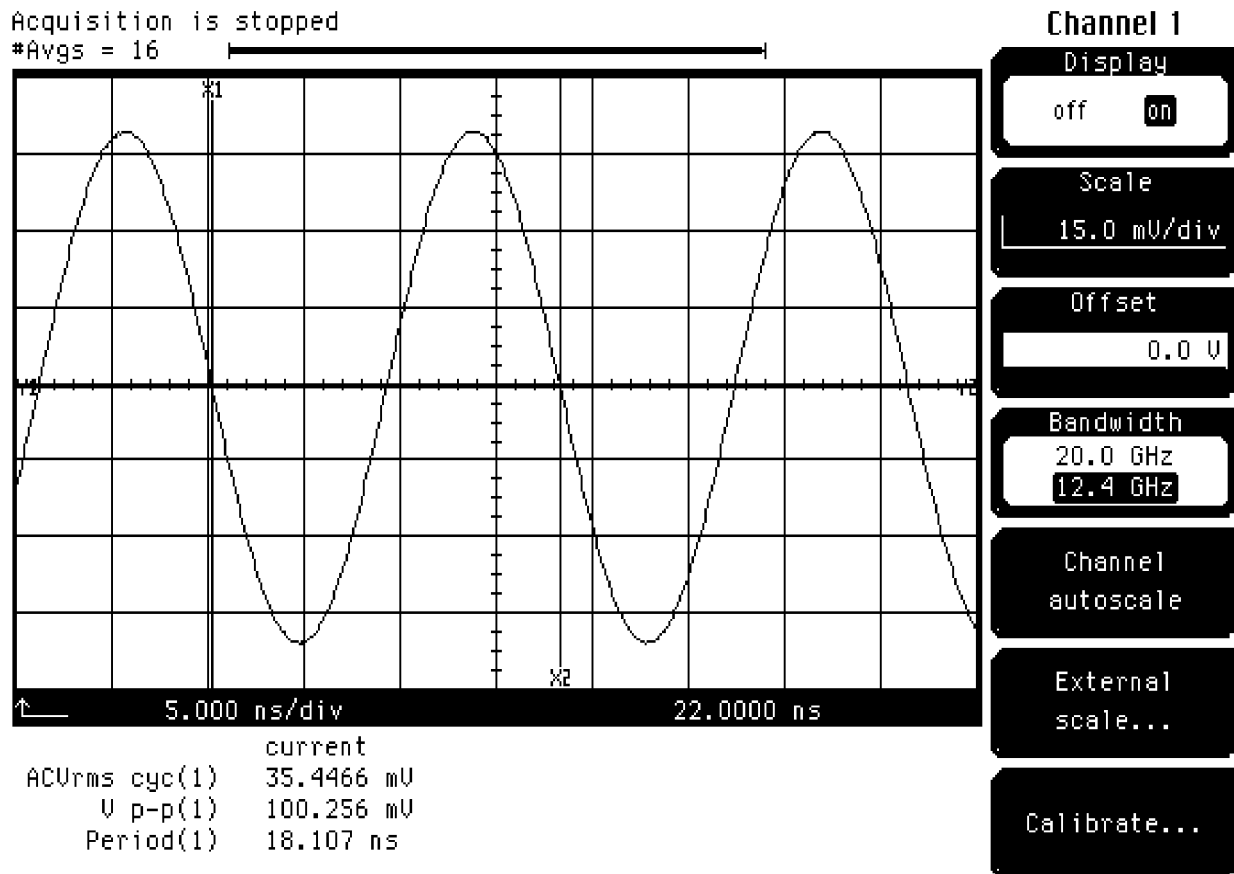
Post-demodulation processing of the (AM-XMOD) sideband energy can also be accomplished in a variety of ways, all of which can be thought of as a ‘detection process’ itself. The most simple and common method is ‘Averaging’. This process can easily be achieved using analog circuit or digital computation methods, depending upon how the demodulated signal energy is formatted (i.e. whether it is discrete data obtained from a digital sampling, or if it is a continuous signal obtained from analog signal processing). In either case, Averaging does not provide the best means of post processing the sideband energy. A superior method, which often provides the best dynamic range and resolution of the sideband spectral energy, is the method of ‘Fast Fourier Transform’ processing. This is a basic digital sampling method, whereby the Spectral Density of the AM-XMOD is analyzed by computational means: the sampled data is used in a digital computation of the Spectral Density of the AM-XMOD. Since the sampling is carried out on signals of low frequencies (i.e. ‘baseband’), sampling speeds can easily be chosen as high as five times the required Nyquist rate and as high as 12 bits per sample (or more) can be used with no loss in sampling fidelity. An instrument that can carry out this function is many times termed a ‘Baseband (Spectrum) Analyzer’, and is readily available as a piece of test equipment.

As the final topic, PM-XMOD effects on the measurement are as follows:

In the derivation of cross modulation it was shown that energy of amplitude modulation could be partly translated to phase modulation. This process is a result of the junction capacitance, as well as the nonlinear current-voltage junction relationship of the device, which together produce additional cross modulation components. The individual cross modulation components are produced in quadrature pairs and form a complex signal. The magnitude of which can be viewed as not only related to the magnitudes of the coefficients of the third order non-linearity, but also to the phase of the third order non-linearity, as a function of frequency. (Note: this was demonstrated as a simple case, in the derivation.) This phase dependence is a result of the capacity of the junction. Therefore, the higher the carrier frequency of a particular channel, the greater the phase of the third order distortion. As well, the larger the frequency separation of the respective channels which produce a given cross modulation component, the greater the phase-difference of the distortion and therefore the greater the magnitude of the PM-XMOD component produced. In short, the higher frequency channels will have cross modulation distortion with greater PM-XMOD components.

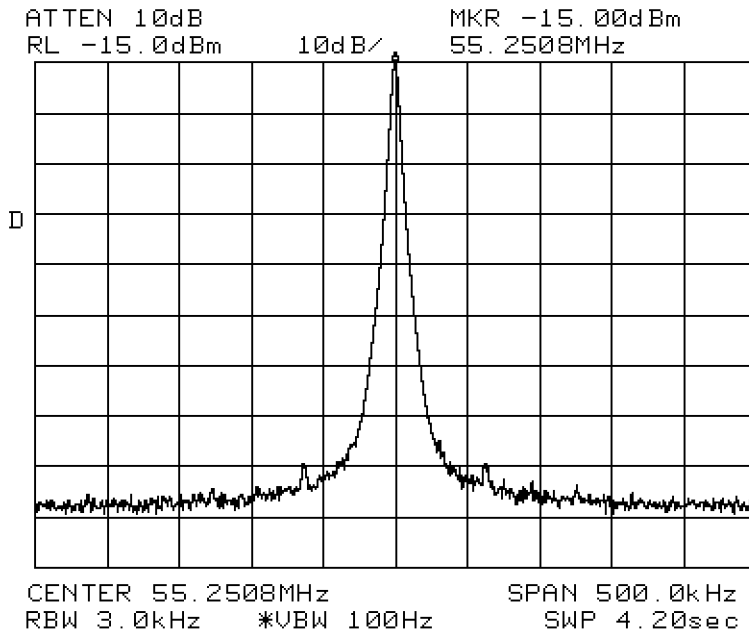
In addition, the composite cross modulation signal, at any given channel, is made up of multiple individual cross modulation components. The phase difference between any of these individual components will also be greater at the higher frequencies, because of the variety of different carrier frequencies that make up these components. Therefore, the composite cross modulation distortion, at the higher carrier frequencies, constitutes a greater variation in the envelope of the composite distortion itself. A ‘peak detector’ will then have a response that has more variation in time, as a repeated measurement, than a SQL detector.

If the Spectrum Analyzer alone is used to measure the (low-level) distortion at these higher frequencies, greater variations in measurement can result because of several reasons. The first is the variations as just previously mentioned, of the detected distortion signal, due to the ‘peak detection’ methods normally employed in Spectrum Analyzers. The second is the greater variations in measurement due to the detector’s response to noise. Both of these can be somewhat overcome by multiple measurements that are averaged. (However, this tends to be a slower measurement process.) The third issue that arises is due to the discrimination of PM by the Resolution Bandwidth filter. This basically is a conversion of PM to AM, by slope detection; that is, if the Resolution Bandwidth filter is set narrow enough, the sideband energy falls on the slope of the filter, which converts PM to AM. Worse, is that the Spectrum Analyzer is operated in a “sweep frequency” mode. Therefore, the rate at which PM is converted to AM is not constant and will give wide variances of the AM-XMOD measurement, with each trace sweep.

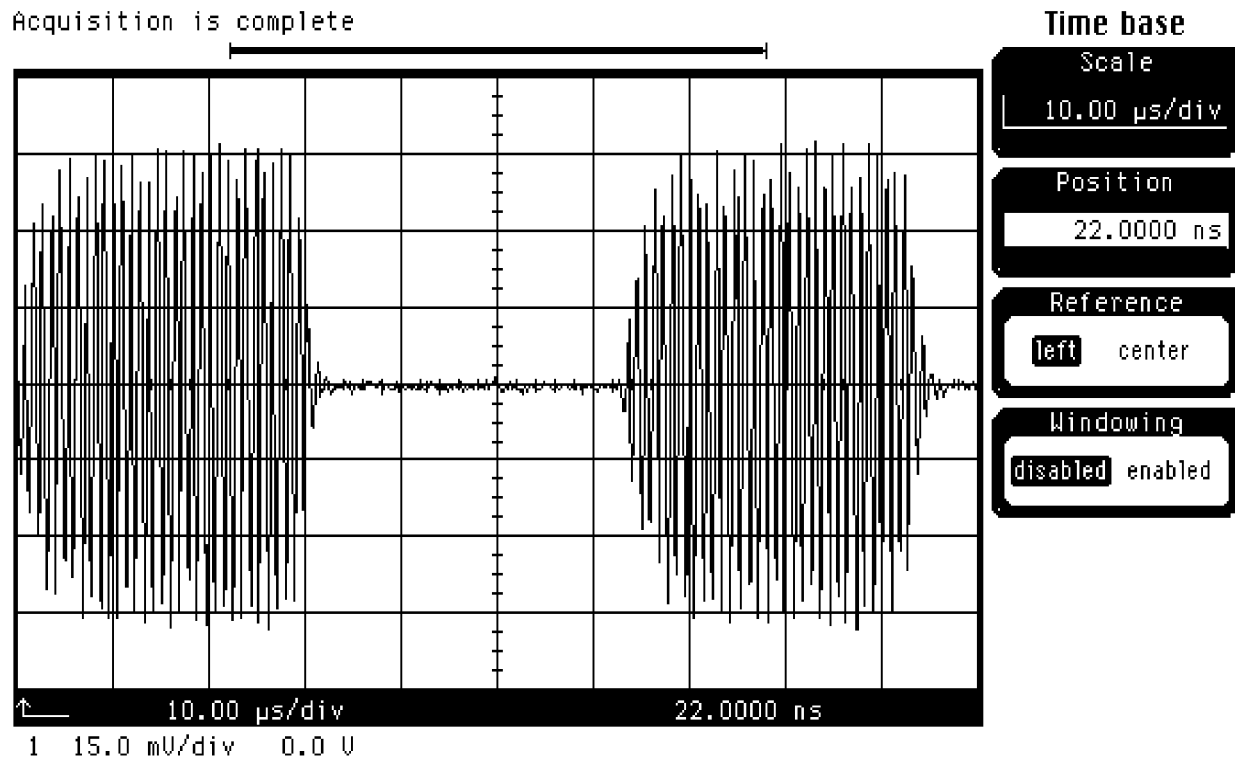


**FIGURE A1**

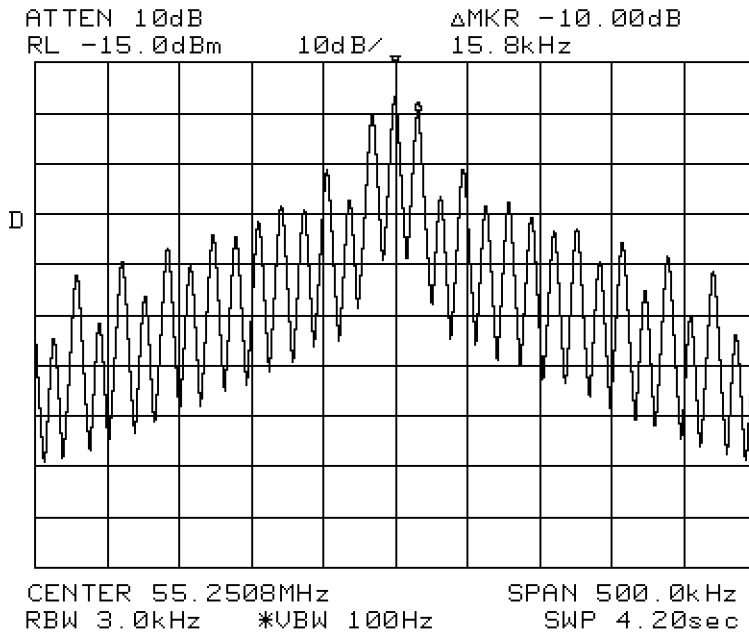




**FIGURE A2**



**FIGURE A3**



**FIGURE A4**

## 9.0 INFORMATIVE REFERENCES

ANSI/SCTE 96 2008: Cable Telecommunications Testing Guidelines

Technical Handbook for CATV Systems; K.A. Simons; Jerrold Electronics Corp., Publication No. 436-001-01, Third Edition; 1968; chapters 4-8.

The Decibel Relationships Between Amplifier Distortion Products; K.A. Simons; Proceedings of the IEEE, Vol. 58, July 1970; pages 1071-1086.

Extending the Limits of Composite Intermodulation Distortion Measurements; Matrix Test Equipment Inc., Matrix Technical Notes, MTN-110, October 4, 1997 (Revised); 14 pages.

Cross modulation and intermodulation in amplifiers at high frequencies; R.G. Meyer, M.J. Shensa and R. Eschenbach; IEEE Journal of Solid State Circuits, Vol. SC-7, Part 1, Feb. 1972; pages 16-23.

Multichannel cross modulation in CATV amplifiers; A. Prochazka; IEEE Transactions on Broadcasting, Vol. BC-20, Part 3, Sept. 1974; pages 33-35.

Intermodulation noise and products due to frequency-dependent nonlinearities in CATV systems; K.Y. Chang et al.; IEEE Transactions on Communications, Vol. COM-2, Part 1, Jan. 1975; pages 142-155.

Phase effects in cross-modulation; L. Gumm; IEEE Transactions on Cable Television, Vol. CATV-1, No. 1, Oct. 1976; pages 5-10.

Cross-Modulation in HRC Systems; G.G. Luettgenau; 30<sup>th</sup> Annual NCTA Convention, Technical Paper; June 1981; pages 67-72.

Characteristics and Perceptibility of Crossmodulation; R. Pidgeon; 32<sup>nd</sup> Annual NCTA Convention, Technical Paper; June 1983; pages 129-134.

Cross modulation: Its specification and significance; R. Pidgeon and M. Adams; 35<sup>th</sup> Annual NCTA Convention, Technical Paper; June 1986; pages 161-165.